Trends in hybrid data tomography

Workshop at DTU Compute Wednesday January 24, 2018
Room 324/050

Program

09:00–09:50 **Stochastic Gradient Descent for Inverse Problems**
Bangti Jin, University College London

09:50–10:40 **Calculating and measuring the current flow during transcranial weak current stimulation**
Axel Thielscher, Danish Research Centre for Magnetic Resonance, and Technical University of Denmark

10:40–11:00 *Coffee*

11:00–11:30 **Stable source reconstruction from finitely many frequency observations in the Multi-frequency Inverse Source Problem**
Adrian Kirkeby,Technical University of Denmark and NTNU

11:30–12:00 **Boundary determination for AET and CDII**
Tommi Brander, Technical University of Denmark

12:00–13:00 *Lunch*

13:00–13:50 **Using multiple frequencies to enforce non-zero constraints in PDE and applications to hybrid inverse problems**
Giovanni S. Alberti, University of Genoa

13:50–14:40 **Quantitative photoacoustic tomography**
Tanja Tarvainen, University of Eastern Finland

14:40–15:00 *Coffee*

15:00–15:30 **Reconstruction Artifacts in Acousto-Electric Tomography**
Kim Knudsen, Technical University of Denmark

15:30–16:00 **Total Variation Regularized Acousto-Electric Tomography with Neumann Conditions**
Bjørn Jensen, Technical University of Denmark

16:00–16:15 *Break*

16:15–17:05 **Small volume fraction expansions with contrast big and small**
Yves Capdeboscq, University of Oxford

17:05–18:00 *Break*

18:00– *Dinner at Ristoranta Bellini*

Everybody is welcome to attend some or all talks
Register for the entire day by emailing Tommi Brander tobr@dtu.dk

1 Abstract of the talks

Stochastic Gradient Descent for Inverse Problems

09:00–09:50, *Bangti Jin*, University College London
In this talk, I will discuss two stochastic algorithms, the randomized Kaczmarz method and the stochastic gradient descent, for solving inverse problems. These two algorithms have been very popular in practice. However, their theory for ill-posed inverse problems remains largely missing. In this talk, I will present some recent results in the direction, from the perspective of inverse problems.

Calculating and measuring the current flow during transcranial weak current stimulation

09:50–10:40, *Axel Thielscher*, Danish Research Centre for Magnetic Resonance, and Technical University of Denmark
Spatial targeting is often only loosely controlled in studies using transcranial direct or alternating current stimulation (TDCS & TACS). This can add to the interindividual variabilty of the observed physiological stimulation effects, reducing the robustness of the method. I will give examples how simulation studies that use the Finite-Element Method (FEM) and anatomically realistic head models can help to improve spatial targeting in TDCS and TACS. For example, I will highlight how the technical parameters of the employed electrode pads can affect the field distribution in the brain. I will further review some of the electrode configurations used in recent dual-site TACS studies, and argue that alternative montages are needed to reach unambiguous conclusions about the brain areas that underlie the reported behavioral stimulation effects.

The accuracy of the field simulations is limited by the uncertainty of the ohmic tissue conductivities. The conductivity values are based on a rather sparse literature with a substantial spread across studies, highlighting the need for validation studies. In the second half of the talk, I will present our initial results of in-vivo measurements of the TACS current flow in the human brain based on Magnetic Resonance Current Density Imaging (MRCDI), and will relate them to the results of recent invasive recordings of the TACS electric field in patients performed by another group.

Stable source reconstruction from finitely many frequency observations in the Multi-frequency Inverse Source Problem

11:00–11:30, *Adrian Kirkeby*, Norwegian University of Science and Technology, and Technical University of Denmark

We consider the multi-frequency inverse source problem for the Helmholtz equation in the plane. A criterion is proved which, given a source from a certain family of finite-dimensional spaces, constructively identifies minimal finite sets of measurement frequencies sufficient for the unique reconstruction of the source. Under an additional, mild criterion on the frequencies, the reconstruction is showed to be stable, in the sense that it avoids measurement data associated with small singular values. To this end we analyze singular systems of source-to-measurement forward operators at different frequencies, and express the criterion for a minimal frequency set in terms of the positive zeros of the Bessel functions of the first kind and integer order and the source domain radius.

Joint work with Mirza Karamehmedovic and Kim Knudsen from DTU.

Boundary determination for AET and CDII

11:30–12:00, *Tommi Brander*, Technical University of Denmark

We recover conductivity at the boundary of a domain from a combination of current or power density interior and Cauchy boundary data, with quite arbitrary boundary data. Depending on the imaging modality, we either recover the conductivity uniquely, or we recover two alternatives, from which we must select the correct one. The argument is elementary and local. More generally, we consider the weighted p-Laplacian as a forward model and interior data $\sigma|\nabla u|^q$ and find out that single measurement specifies the boundary conductivity when $p - q \geq 1$, and otherwise the measurement specifies two alternatives. There are special problems when $p - q = 1$ and this is also the only case where single interior measurement in one dimension does not specify the conductivity on the interval.

Recvery of conductivity in the interior can be reduced to solving a weighted $p - q$ -Laplace equation. The hyperbolicity of this equation is related to the nonuniqueness of boundary determination when $p - q < 1$.

Joint work with Changyou Li (Northwestern Polytechnical University).

Using multiple frequencies to enforce non-zero constraints in PDE and applications to hybrid inverse problems

13:00–13:50, *Giovanni S. Alberti*, University of Genoa
In this talk I will describe a multiple frequency approach to the boundary control of Helmholtz and Maxwell equations. We give boundary conditions and a finite number of frequencies such that the corresponding solutions satisfy certain non-zero constraints inside the domain. The suitable boundary conditions and frequencies are explicitly constructed and do not depend on the coefficients, in contrast to the illuminations given as traces of complex geometric optics solutions.

This theory finds applications in several hybrid imaging modalities: these constraints are needed to prove stability and to apply explicit reconstruction formulae. Similarly, multiple frequencies guarantee uniqueness and stability for the linearised inverse problem in acousto-electromagnetic tomography, thereby obtaining the convergence of a Landweber iteration scheme.

Quantitative photoacoustic tomography

13:50–14:40, *Tanja Tarvainen*, University of Eastern Finland
Photoacoustic tomography is a hybrid imaging modality which is based on photoacoustic effect caused by the absorption of an externally introduced light pulse in the medium. The technique combines optical contrast with high spatial resolution of ultrasound. In the inverse problem of quantitative photoacoustic tomography, one aims at estimating the distribution of optical parameters from the photoacoustic time-series measured on the surface of the target. Often, this is approached in two phases: first by estimating the initial pressure caused by the absorption and after that by estimating the distribution of absorbers from the reconstructed initial pressure. In this talk, the inverse problem of quantitative photoacoustic tomography is discussed. The problem is approached in the framework of Bayesian inverse problems.

Reconstruction Artifacts in Acousto-Electric Tomography

15:00–15:30, *Kim Knudsen*, Technical University of Denmark

Total Variation Regularized Acousto-Electric Tomography with Neumann Conditions

15:30–16:00, *Bjørn Jensen*, Technical University of Denmark

Small volume fraction expansions with contrast big and small

16:15–17:05, *Yves Capdeboscq*, University of Oxford
In this talk I will discuss recent progresses in the understanding of asymptotic developments for solutions of elliptic second order boundary value problems where the parameters are perturbed on an area of small volume fraction. I will discuss recent progresses related to the presence of large contrasts.